Exercise of 'Theory of Firm', June 16th, 2016

1. コブダグラス型技術に関して以下を導出せよ.

Regarding to the Cobb-Douglass type technology, find followings.

(a) 長期の費用関数 $c(\boldsymbol{w}, y)$ を求めよ.

Find a long-run cost function $c(\boldsymbol{w}, y)$ by solving the following problem.

$$c(\mathbf{w}, y) = \min_{x_1, x_2} w_1 x_1 + w_2 x_2$$

s.t. $x_1^a x_2^{1-a} = y$

(b) 短期の費用関数 $c(\boldsymbol{w}, y, k)$ を求めよ (k は固定要素).

Suppose that k is the fixed amount of factor input of good 2. $(x_2 = k)$. Find a short-run cost function $c(\boldsymbol{w}, y, k)$ by solving the following problem.

$$c(\mathbf{w}, y, k) = \min_{x_1} w_1 x_1 + w_2 k$$

s.t. $x_1^a k^{1-a} = y$

(c) 短期の利潤関数 $\pi(p, \boldsymbol{w}, k)$ を求めよ (k は固定要素).

Find a short-run profit function $\pi(p, \boldsymbol{w}, k)$ by solving the following problem.

$$\pi(p, \boldsymbol{w}, k) = \max_{x_1, y} py - w_1 x_1 - w_2 k$$

s.t. $x_1^a k^{1-a} = y$

2. ある財を y 単位生産し、資本整備水準 k で操業している企業の短期費用関数が

$$c(y,k) = 8y^3k^{-2} + 2k$$

で表わされるものとする.企業は長期において、資本整備の大きさを調整費用を払うことなしに変更できるとする.

- (a) 資本整備が k=2 に固定され、財の市場価格が 24 であるとき、企業の (短期における) 供給量 y を求めよ.
- (b) 長期における最適な資本整備水準 k(y) を求めよ.
- (c) この企業の長期費用関数 c(y) を求めよ.

Suppose that the short-run cost function of a firm which produces the output y with capital input k, is represented by the following equation.

$$c(y,k) = 8y^3k^{-2} + 2k$$

Assume that this firm can change the amount of capital input without any cost in the long-run.

- (a) Derive the amount of product y when the capital input is fixed as k=2 and the price of output is 24.
- (b) Find the optimal capital input k(y) in the long-run.
- (c) Derive the long-run cost function c(y).
- 3. (短期における) 企業の総費用関数が

$$c(y) = \frac{1}{3}y^3 - 2y^2 + 5y + \frac{32}{3}$$

で表されるとき,以下の問いに答えなさい.

- (a) 限界費用 (MC), 平均費用 (AC), 平均可変費用 (AVC) を求め, グラフの概略 を図示せよ.
- (b) 損益分岐点及び操業停止点を求め、グラフ上で各点を指摘せよ.
- (c) 財の市場価格をpで表す.この企業の供給関数y(p)を求めなさい.

Suppose the (short-run) cost function of a firm is represented by the following equation.

$$c(y) = \frac{1}{3}y^3 - 2y^2 + 5y + \frac{32}{3}$$

- (a) Calculate Marginal Cost (MC), Average Cost (AC) and Average Variable Cost (AVC). Then, illustrate these functions.
- (b) Find the break-even point and the shutdown point. Then, draw these points on the graph.
- (c) Let p denote the market price of the good. Find the supply function y(p) of this firm.